2021년 4月 학평 수학 시험.

#1. ①

$(2x+y)+i(x-y)=1+2i.$

$2x+y=1$

$x-y=2$

$3x=3 \Leftrightarrow x=1$

$y=-1$

$\therefore (x+yi)^4$

$=(1-i)^4$

$=\{(1-i)^2\}^2$

$=(-2i)^2$

$=-4.$

#2. ②

x^3+3x^2+x+2

$=(x^2+x-1)x+2x^2+2x+2$

$=(x^2+x-1)\times 2+4$

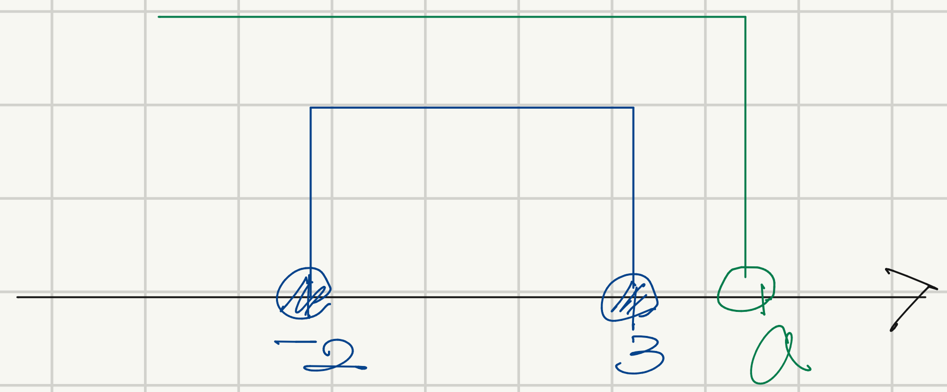
$=4.$

#3. ④

$p : x^2-x-6\le 0$ $\quad P=\{x \mid -2\le x\le 3\}$

$q : x<a$

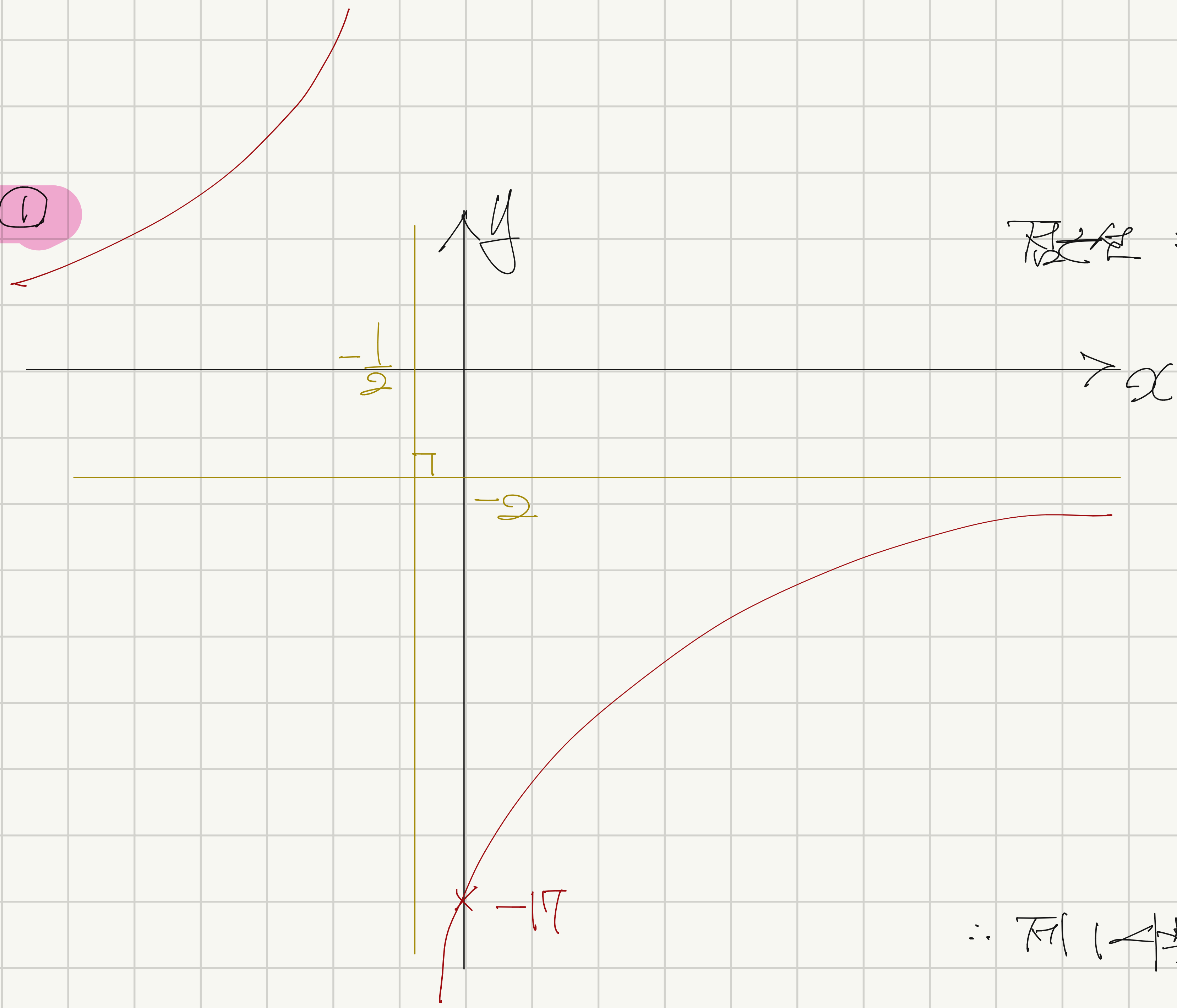
$p\Rightarrow q$ 이므로



$3<a$

∴ 정수 a의 최솟값은 4.

#4. ①

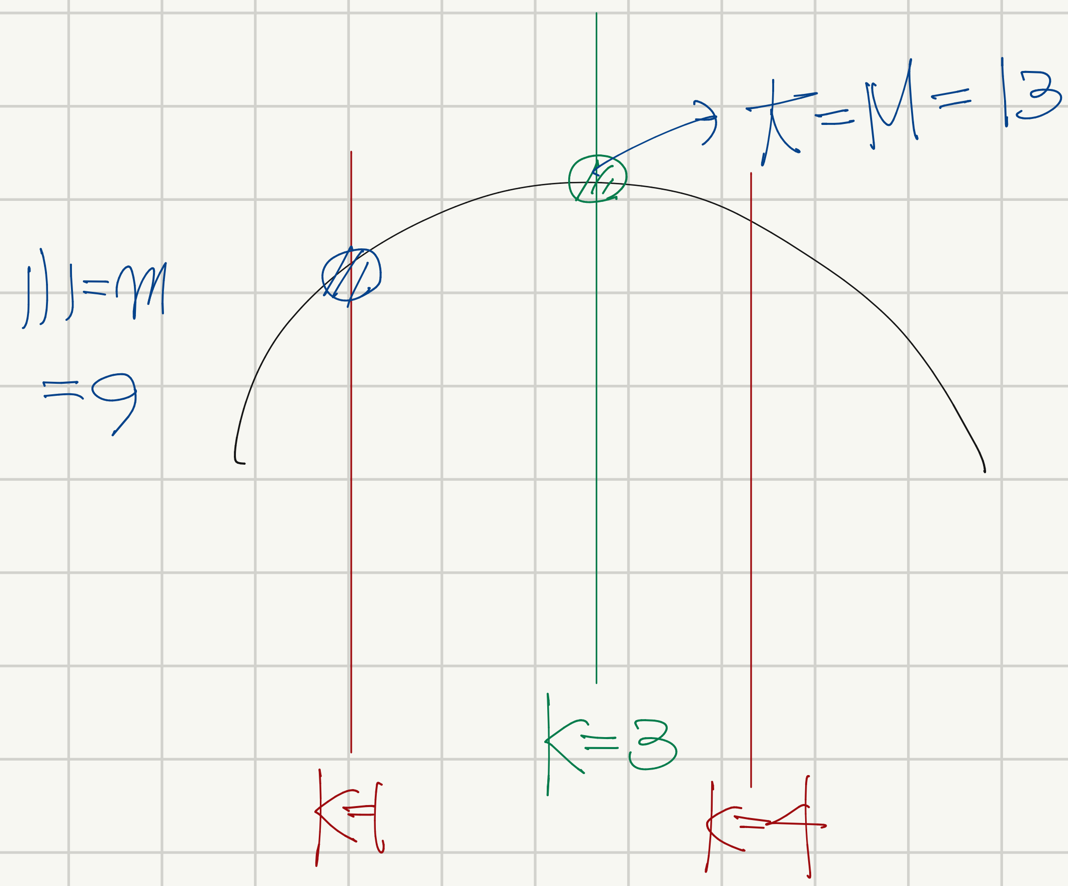


점근선 : $x=-\frac{1}{2}$, $y=-2$

∴ 제 1사분면은 지나지 않는다.

#5. ④ $\alpha+\beta=3k$, $\alpha\beta=-k^2$ 이므로

$$(\alpha+2)(\beta+2)=\alpha\beta+2(\alpha+\beta)+4$$
$$=-k^2+6k+4$$



$\therefore$ $M-m$
$=13-9$
$=\boxed{4}$

#6. ②

$\text{小}=f(-2)=\left(\frac{1}{2}\right)^{2+a}+b=\frac{41}{8}$

$\text{大}=f(2)=\left(\frac{1}{2}\right)^{-2+a}+b=7$

두 식을 빼면 $\left(\frac{1}{2}\right)^{2+a}-\left(\frac{1}{2}\right)^{-2+a}=-\frac{15}{8}$.

$\left(\frac{1}{2}\right)^{a}=t\ (t>0)$ 이라 치환하자.

$$\frac{1}{4}t-4t=-\frac{15}{8}$$

$$-\frac{15}{4}t=-\frac{15}{8} \Leftrightarrow t=\frac{1}{2}$$

$\left(\frac{1}{2}\right)^{a} = \frac{1}{2} \iff a=1,\ b=5$

$\therefore a+b = \boxed{6}$

기출

#17. ②

분모, 분자를 유리화하면

$$\lim_{x\to 0} \frac{8(\sqrt{2x+3}+\sqrt{2x+1})}{2(\sqrt{x+9}+\sqrt{x+1})}$$

$$= 4 \times \frac{2\sqrt{2}}{2}$$

$$= 4\sqrt{2}.$$

#18. ②

$P(B|A)=P(B) \iff A, B$는 독립.

$P(A)-P(B)=\frac{1}{6} \iff P(A)=\frac{1}{6}+P(B)$

$P(A\cap B)=P(A)P(B)=\frac{1}{6}.$

대입.

$\left(\frac{1}{6}+P(B)\right)P(B)=\frac{1}{6}.$

$$\{P(B)\}^2 + \frac{1}{6}P(B) - \frac{1}{6} = 0$$

$$6\{P(B)\}^2 + P(B) - 1 = 0$$

$$\begin{array}{ccc} 3P(B) & & -1 \\ 2P(B) & & 1 \end{array}$$

$$\therefore P(B) = \frac{1}{3}.$$

#9. ①

두 함수는 역함수 관계!

$$x^2 - 12 = x$$

$$x^2 - x - 12 = 0$$

$$\begin{array}{c} -4 \\ 3 \end{array}$$

$x \geq 0$ 이므로 $x = 4$.

$$\therefore (a, b) = (4, 4)$$

$$a + b = 4 + 4 = 8.$$

#10. ①

$$S_4 - S_2 = a_2^2$$

$$\Leftrightarrow a_3 + a_4 = a_2^2$$

$$\Leftrightarrow ar^2 + ar^3 = (ar)^2$$

$$1 + r = a$$

$$a = \frac{1}{3}, \quad r = -\frac{2}{3}.$$

$$S_4 = \frac{\frac{1}{3}\left\{1 - \left(-\frac{2}{3}\right)^4\right\}}{1 - \left(-\frac{2}{3}\right)}$$

$$= \frac{1}{5}\left(1 - \frac{16}{81}\right)$$

$$= \frac{1}{5} \times \frac{65}{81}$$

$$= \frac{13}{81}$$

$$\therefore q = 13.$$

개꿀.

#11. ③

$$\frac{{}_3C_1 \times {}_4C_1}{{}_7C_2} = \frac{12}{21} = \frac{4}{7}.$$

#12. ②

(주기) $= 4\pi = \dfrac{2\pi}{|b|} \iff b = \dfrac{1}{2}$.

최대 $= 4 = 3 + C \iff C = 1$

$\therefore\ b + C = \dfrac{1}{2} + 1$

$= \dfrac{3}{2}$.

#13. ③

(진수) > 0 이므로

$2x-1>0,\ 8-x>0 \iff \dfrac{1}{2} < x < 8$

$\log_{\frac{1}{2}}(2x-1) < \log_{\frac{1}{2}}\dfrac{1}{2} + \log_{\frac{1}{2}}(8-x) = \log_{\frac{1}{2}}\dfrac{1}{2}(8-x)$

$2x-1 > \dfrac{1}{2}(8-x)$

$4x-2 > 8-x$

$5x > 10 \qquad x>2$

$\therefore\ 2<x<8$

따라서 x는 3, 4, 5, 6, 7 이므로 5개.

#14. ①

$$\lim_{x\to 1} \frac{x^2 f(1) - f(x)}{x-1} \doteq \frac{0}{0} \text{ 꼴이므로}$$

$$= \lim_{x\to 1} \left(2x f(1) - f'(x)\right)$$

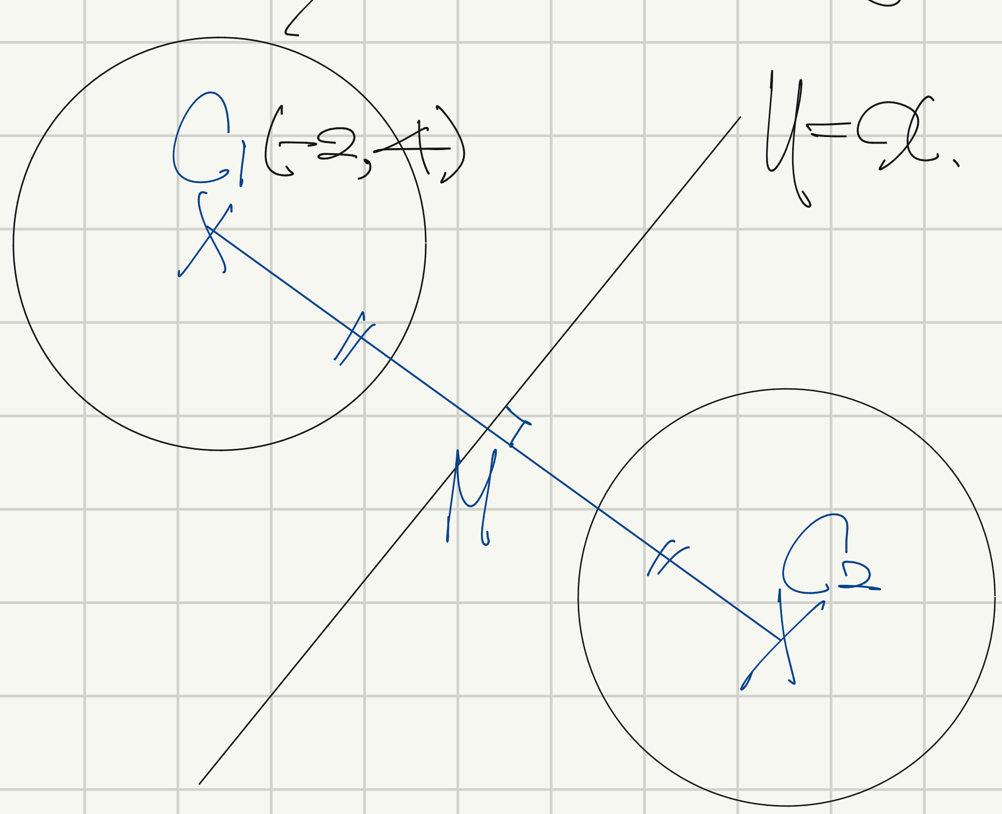
$$= 2f(1) - f'(1).$$

$f(x) = x^3 - 2x^2 + 4x - 1 \rightarrow f(1) = 2.$

$f'(x) = 3x^2 - 4x + 4 \rightarrow f'(1) = 3.$

$\therefore\ 2 \times 2 - 3 = 1.$

#15. ④ $\rightarrow (x+2)^2 + (y-4)^2 = 25$



$\overline{C_1C_2} = 2\overline{C_1M}$

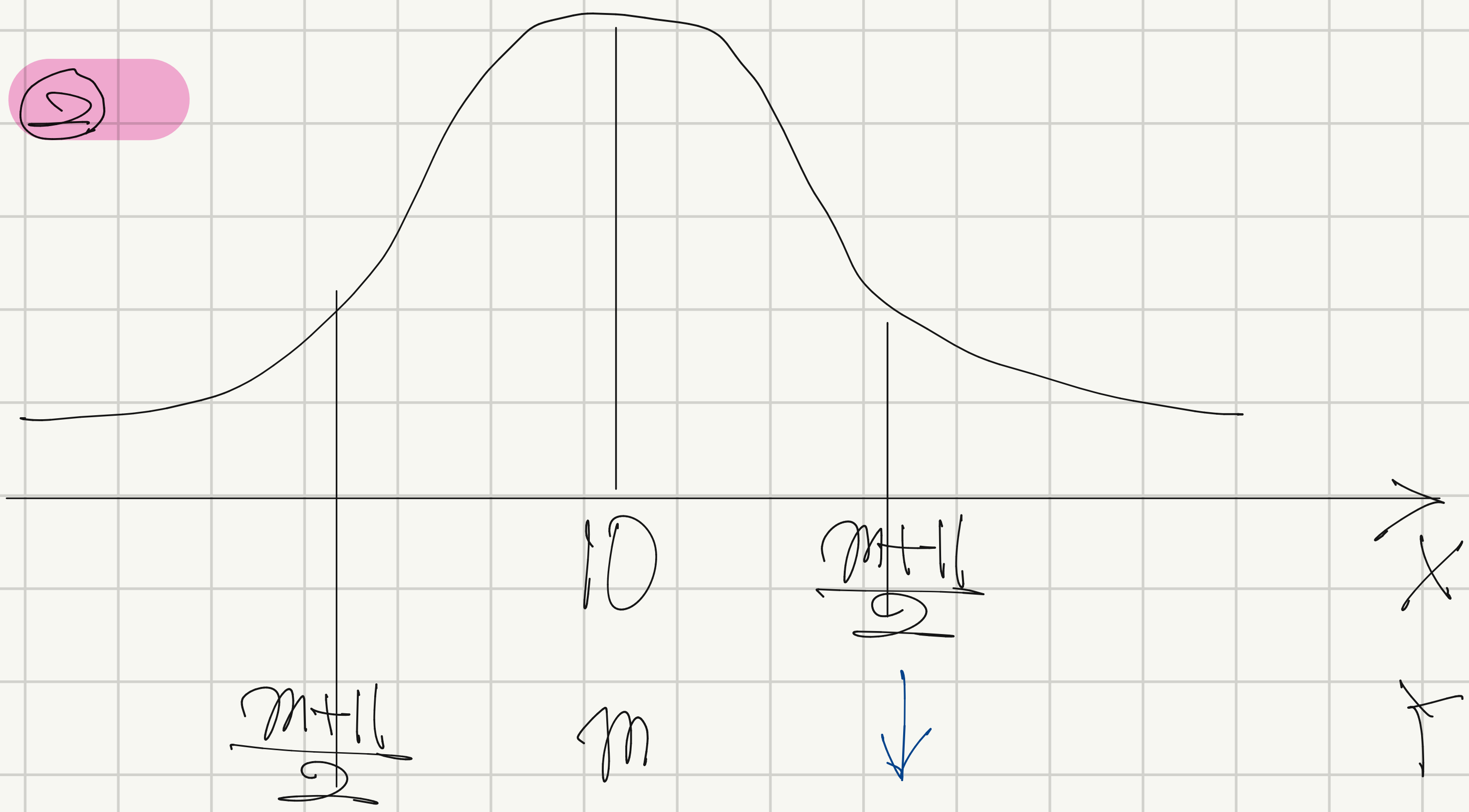
$(-2,4) \sim x-y=0$

$$\Rightarrow \frac{|-2-4|}{\sqrt{1+1}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}.$$

$\therefore 6\sqrt{2}.$

기출

#16. ②



$$\frac{\frac{m+11}{2}-m}{2} \qquad \frac{\frac{m+11}{2}-10}{3}$$

합이 0 이다!

양변에 12를 곱하면

$$3m+33-6m+2m+22-40=0$$

$\therefore m=15.$

#17. ②

$$\lim_{x \to 1-} (x+1) = 2$$

$$\lim_{x \to 1+} (3ax-b) = 3a-b$$

$\underline{3a-b=2}$ ⋯ ㉠

$$\lim_{x \to 3-} (3ax-b) = 9a-b$$

$$\lim_{x \to 3+} (bx+a) = 3b+a$$

$9a-b = 3b+a$

$\Leftrightarrow \underline{2a=b}$ ⋯ ㉡

㉠, ㉡을 연립하면

$a=2,\ b=4$ 이다.

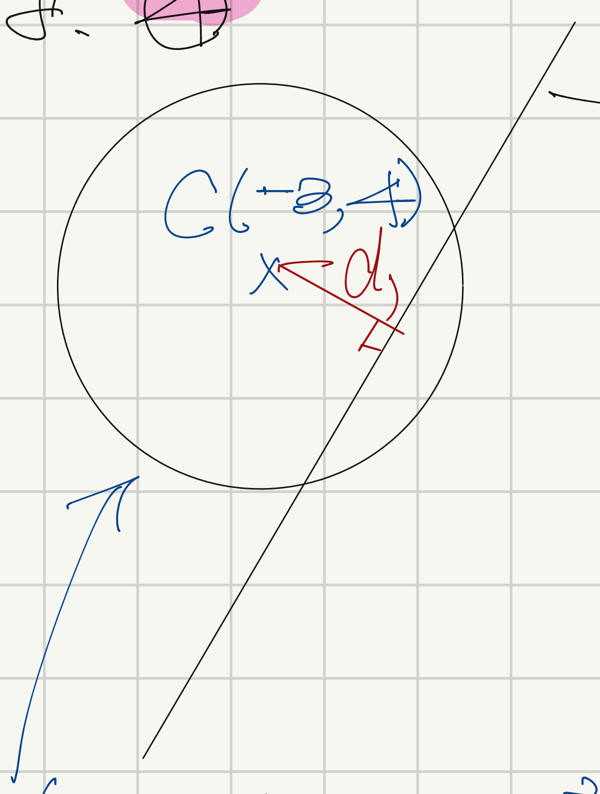
$$\int_{-2}^{3} (2x+4)\,dx$$

$$= \left[x^2+4x\right]_{-2}^{3}$$

$$= 5+4\cdot 5$$

$$= 25.$$

#18. ④



→ $2x+y+a=0$

$(x+3)^2+(y-4)^2=5.$

$d \le \sqrt{5}$ 이어야 원과 직선이 만난다.

d: $C(-3,4) \sim 2x+y+a=0$

$$d = \frac{|a-2|}{\sqrt{5}} \le \sqrt{5}$$

$$|a-2| \le 5$$

$$-5 \le a-2 \le 5$$

$$-3 \le a \le 7.$$

∴ 실수 a의 최댓값은 7이다.

#19. ②

$t^2-2t+3 = f(t)$라 하자.

$$\lim_{x\to3}\frac{F(x)-F(3)}{x^2-9}$$

$$=\lim_{x\to3}\frac{F(x)-F(3)}{x-3}\times\frac{1}{x+3}$$

$$=F'(3)\times\frac{1}{6}$$

$$=\frac{1}{6}\times f(3)$$

$$=\frac{1}{6}\times(9-6+3)$$

$$=1.$$

#20. ③

$h'(x)=f'(x)-g'(x).$

h가 극대인 점 $x=a$라 하면

$x=a$를 기준으로 h' 값이 양수에서 음수로 바뀌어야 한다.

즉, f'이 g'보다 컸다가 작아지는 점이어야 한다.

$\therefore x=2.$