

2019년 제3차 경찰공무원(해경) 수학 해설

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1. 【정답】 ③

$$3\sqrt{4} + n = 0, \quad n = -6$$

2. 【정답】 ②

$$2^2 = 4 \text{개다.}$$

3. 【정답】 ①

$$E(X) = b + 2c = 1$$

$$V(X) = E(X^2) - \{E(X)\}^2 = b + 4c - (b + 2c)^2 = \frac{1}{2}$$

$$2c = \frac{1}{2}, \quad c = \frac{1}{4}, \quad b = \frac{1}{2}, \quad a = \frac{1}{4}$$

$$abc = \frac{1}{4} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{32}$$

4. 【정답】 ④

$$a + 2b = -c, \quad a^2 + 4ab + 4b^2 = c^2$$

$$-a^2 - 4b^2 + c^2 + 4ab = -a^2 - 4b^2 + a^2 + 4ab + 4b^2 + 4ab = 8ab$$

5. 【정답】 ④

① 수열 1, 3, 9, 27, 81, ...의 일반항은 3^{n-1} 이다. (단, n 은 자연수)

$$\textcircled{2} \sum_{k=1}^{2n} 4 = 8n$$

$$\textcircled{3} \sum_{k=1}^{100} k^2 = \frac{100 \times 101 \times 201}{6}$$

$$\textcircled{4} 1^3 + 2^3 + 3^3 + \cdots + n^3 = \sum_{k=1}^n k^3 = \left\{ \frac{n(n+1)}{2} \right\}^2 = (1 + 2 + 3 + \cdots + n)^2$$

6. 【정답】 ③

$$\lim_{h \rightarrow 0} \frac{f(a-3h) - f(a)}{h} = \lim_{h \rightarrow 0} \frac{f(a-3h) - f(a)}{-3h} (-3) = -3f'(a)$$

7. 【정답】 ②

$$\int_0^3 xf'(x)dx = k \text{라 하면 } f(x) = 4x + k$$

$$\int_0^3 xf'(x)dx = \int_0^3 4xdx = [2x^2]_0^3 = 18$$

$$f(x) = 4x + 18, f(-3) = -12 + 18 = 6$$

8. 【정답】 ④

$$x \rightarrow 0+ \text{일 때 } f(x) \rightarrow 3-0 \text{이므로 } \lim_{x \rightarrow 0} f(f(x)) = 3$$

$$x \rightarrow 2+ \text{일 때 } f(x) = 3 \text{이므로 } \lim_{x \rightarrow 0} f(f(x)) = 2$$

$$3 + 2 = 5$$

9. 【정답】 ②

$$f(x) = x^5 + 2x^2 + k - 3 \text{이라 하면}$$

$$f(1) = 1 + 2 + k - 3 = k$$

$$f(2) = 2^5 + 2^3 + k - 3 = k + 37$$

$$f(1)f(2) < 0 \text{이면 구간 } (1, 2) \text{에서 적어도 하나의 실근을 가지므로}$$

$$k(k+37) < 0, -37 < k < 0$$

$$\text{따라서 정수 } k \text{의 개수는 } 37 - 1 = 36 \text{개다.}$$

10. 【정답】 ①

$${}_7C_4 a^3 = 280, {}_{35}a^3 = 280, a = 2$$

$$x^5 \text{의 계수는 } {}_7C_5 \cdot 2^2 = 84$$

11. 【정답】 ③

$$A, B \text{ 모두 뽑히지 않을 확률 } \frac{{}_6C_5}{{}_8C_5} = \frac{6}{56} = \frac{3}{28} \text{이므로}$$

$$\text{따라서 문제의 확률은 } 1 - \frac{3}{28} = \frac{25}{28}$$

12. 【정답】 ④

$$\text{원점 } O \text{와 직선 } \sqrt{3}x + y = k \text{ 사이의 거리 } d = \frac{|-k|}{2} = \frac{|k|}{2}$$

$$\text{이때 } \overline{AB} = 2\sqrt{1^2 - \left(\frac{|k|}{2}\right)^2} = 2\sqrt{1 - \frac{k^2}{4}} = \sqrt{4 - k^2}$$

$$\text{삼각형의 넓이 } S = \frac{1}{2} \sqrt{4-k^2} \times \frac{|k|}{2} = \frac{1}{4} \sqrt{4k^2-k^4}$$

$$f(k) = 4k^2 - k^4 \text{이라 하면 } f'(k) = 8k - 4k^3 = 0, k^2 = 2$$

13. 【정답】 ②

3의 배수의 눈이 나온 횟수를 a 라 하면

$$\sqrt{a^2 + (4-a)^2} \leq 3$$

$$a^2 + 16 - 8a + a^2 \leq 9$$

$$2a^2 - 8a + 7 \leq 0$$

$$\frac{4-\sqrt{2}}{2} \leq a \leq \frac{4+\sqrt{2}}{2} \text{이므로 } a = 2 \text{이다.}$$

$${}_4C_2 \left(\frac{2}{6}\right)^2 \left(\frac{4}{6}\right)^2 = \frac{8}{27}$$

14. 【정답】 ①

$$x-2 > 0, x-2 \neq 1$$

$$x > 2, x \neq 3$$

$$-x^2 + 7x - 6 > 0, x^2 - 7x + 6 < 0$$

$$(x-1)(x-6) < 0, 1 < x < 6$$

최종 범위는 $2 < x < 6, x \neq 3$ 이므로

$$\text{모든 정수 } x \text{의 합은 } 4+5=9$$

15. 【정답】 ④

$$x^2 + 3x + a + (-3x - 6)i = 0$$

$$-3x - 6 = 0, x = -2$$

$$4 - 6 + a = 0, a = 2$$

16. 【정답】 ②

$$(2[x] - 1)(2[x] - 5) \leq 0$$

$$\frac{1}{2} \leq [x] \leq \frac{5}{2} \text{이므로 } [x] = 1, 2 \text{이다.}$$

$$\text{따라서 } 1 \leq x < 3 \text{이므로 } \alpha^2 + \beta^2 = 1^2 + 3^2 = 10$$

17. 【정답】 ①

정규분포 $N(350, 16^2)$ 을 따르므로 64개를 임의 추출한 표본은 정규분포 $N(350, 2^2)$ 를 따른다.

$$\begin{aligned} P(346 \leq X \leq 355) &= P\left(\frac{346-350}{2} \leq Z \leq \frac{355-350}{2}\right) = P(-2 \leq Z \leq 2.5) \\ &= 0.4772 + 0.4938 = 0.9710 \\ 1 - 0.9710 &= 0.0290 \end{aligned}$$

18. 【정답】 ③

$$\begin{aligned} S_n &= \int_0^1 (x^n - x^{n+1}) dx = \left[\frac{1}{n+1} x^{n+1} - \frac{1}{n+2} x^{n+2} \right]_0^1 = \frac{1}{n+1} - \frac{1}{n+2} \\ \sum_{k=1}^{100} \left(\frac{1}{k+1} - \frac{1}{k+2} \right) &= \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{101} - \frac{1}{102} = \frac{1}{2} - \frac{1}{102} = \frac{100}{204} = \frac{25}{51} \end{aligned}$$

19. 【정답】 ④

$$\begin{aligned} \sum_{k=1}^{255} \log_2 \left(1 + \frac{1}{k} \right) &= \log_2 \left(\frac{2}{1} \right) + \log_2 \left(\frac{3}{2} \right) + \dots + \log_2 \left(\frac{256}{255} \right) = \log_2 \left(\frac{2 \times 3 \times \dots \times 256}{1 \times 2 \times \dots \times 255} \right) \\ &= \log_2 256 = 8 \end{aligned}$$

20. 【정답】 ①

$$\int_0^2 ax dx + \int_2^3 (-2ax + 6a) dx = 1$$

$$\left[\frac{a}{2} x^2 \right]_0^2 + \left[-ax^2 + 6ax \right]_2^3 = 2a - 5a + 6a = 3a = 1, \quad a = \frac{1}{3}$$

$$P(1 \leq X \leq 2) = \int_1^2 \frac{1}{3} x dx = \left[\frac{1}{6} x^2 \right]_1^2 = \frac{3}{6} = \frac{1}{2}$$