1. 고차 방정식 $x^3-1=0$의 근의 성질 $x^2+x+1=0$ 의 한근 $x=\frac{-1+\sqrt{3n^1}}{2}$ 을 이용하는 문제.

$x^{10}+x^{20}$의 값을 구하는 문제와 같다.

$x^3=1$이므로 $(x^3)^3.x+(x^3)^6.x^2$=$x+x^2$는 x^2+x+1에서 =-1

[정답] ①

2. 나머지 정리.

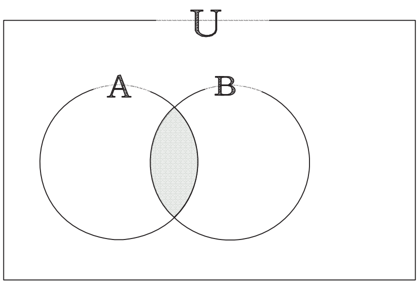
$p(x)=(x-1)(\quad)+3$
$\quad\;\; =(x-2)(\quad)+6 \to p(2)=6$

$p(x)=(x-1)(x-2)(\quad)+a(x-1)+3$
$p(2)=a+3=6 \quad a=3$

나머지 $3(x-1)+3=3x$
$p(3)=9$

[정답] ③

3. 집합의 원소=>벤다이어그램



$A^C \cap B^C = 3,4$

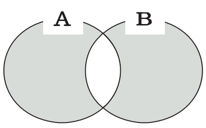
$\hookrightarrow (A\cup B)^C$

$A^C \cap B^C = \{1,2,3,4\}$

$(A\cap B)^C = \{1,2,3,4\}$

$A\cap B=\{5,6,7\}$

$(A-B)\cup(B-A)=$



$\{1,2,5,6,7\}-\{5,6,7\}$

$=\{1,2\}\to$원소합

[정답] ①

4. 다항 함수 미분에 관한 문제.

$f_{(x)}=ax^2+3x+b, f^c_{(x)}=2ax+3$

$f_{(-1)}=a-3+b=3, f^1_{(-1)}=-2a+3=-1$

$a=2, b=4$ ※$f_{(2)}=4a+6+b=18$

[정답] ④

5. 원의 평행이동=>중심의 이동으로 바꾸어 생각.

$x^2+y^2-4x+2y=0 \to x\to 3, y\to 2$

평행이동후 $(x-a)^2+(y-b)^2=c$

$a+b+c=$

(1) $(x-2)^2+(y+1)^2=5$

$(2,-1)\quad r=\sqrt{5}$

$(2-1)\to(2+3,\ -1+2)$
$\hookrightarrow a \quad \hookrightarrow b$

$a=5, b=1, c=5$

$a+b+c=11$

[정답] ④

6. 로그의 값.

↗정수부분
$\log 20 = n+d$
↘소수부분

2자리수=$n+1\to n=1$

$\alpha=\log 20-1=\log 2$

$2^{\frac{1}{n}}+2^{\frac{1}{a}}$

$=2^1+2^{\frac{1}{\log 2}\to\frac{\log 10}{\log 2}=\log 2^{10}}$

$=2+2^{\log 2^{10}}=2+10=12$

[정답] ②

7.

$$\lim_{n\to\infty}(2n+1)an=3$$

$$\lim_{n\to\infty}\frac{b_n}{n^2+1}=2$$

$$\lim_{n\to0}\frac{a_nb_n}{n}=\lim_{n\to0}\frac{(2n+1)an}{n(2n+1)}$$

$$\frac{b_n}{n^2+1}.(n^2+1)$$

$$=\lim_{n\to\infty}\frac{6(n^2+1)}{n(2n+1)}=\frac{6}{2}=3$$

편법=> $a_n=\dfrac{3}{2n+1},\ b_n=2(n^2+1)$

대입 후=> 극한계산

8. 이산 확률 분포$\left(E_{(x)}=2a+\dfrac{3}{2}+6b=5\right)$

$$\begin{cases}a+b=\dfrac{5}{8}\\ 2a+6b=\dfrac{7}{2}\end{cases}$$

$$2a+6b=\frac{5}{4}$$
$$-2a+6b=\frac{7}{2}$$

$$-4b=-\frac{9}{4}$$

$$b=\frac{9}{16},\ a=\frac{5}{8}-\frac{9}{16}$$

$$=\frac{1}{16}$$

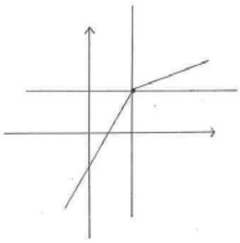
$$b-a=\frac{9}{16}-\frac{1}{16}=\frac{1}{2}$$

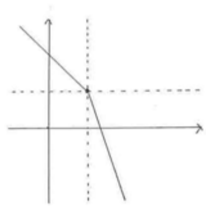
[정답] ①

9. 1:1대응.

$$f_{(x)}=|x-1|-mx+4$$

$$\begin{cases}x\ge1 & f(x)=(1-m)x+3\\ x<1 & f(x)=(-1-m)x+5\end{cases}$$





<기울기 모두양> 또는 <기울기 모두음>

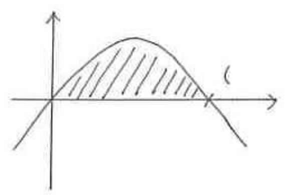
$1-m>0,\ -1-m>0$ $\qquad$ $1-m<0,\ -1-m<0$

$\therefore 1>m,\ -1>m$ $\qquad$ $1<m,\ -1<m$

$\therefore -1>m$ 이거나 $m>1$

[정답] ①

10. $y=\left(\dfrac{1}{2}\right)^n x-x^2$



$$A_n=\int_0^{\left(\frac{1}{2}\right)^n}-x\left(x-\left(\frac{1}{2}\right)^n\right)dx$$

$$=\frac{1}{6}\left\{\left(\frac{1}{2}\right)^n-0\right\}^3$$

$$=\frac{1}{6}.\left(\frac{1}{8}\right)^n$$

$$=\frac{1}{48}\left(\frac{1}{8}\right)^{n-1}$$

$$\therefore\sum_{n=1}^{\infty}An=\frac{\frac{1}{48}}{1-\frac{1}{8}}=\frac{1}{48-6}=\frac{1}{42}$$

[정답] ①

적용공식 1. $\displaystyle\int_\alpha^\beta a(x-\alpha)(x-\beta)dx=-\frac{a}{b}(\beta-\alpha)^3$

2. $|\Upsilon|<1 \quad s=\dfrac{a}{1-\Upsilon}$

3. $a_m=ar^{n-1}$(등비r,초항a)

11. 다항식 연산(동류항 계산)

$2A+B=8x^2+3xy-12y^2$ -①
$+A-B=x^2-7y^2$ -②

$3A=9x^2+3xy-12y^2$

$A=3x^2+xy-4y^2$

$2A+B=8x^2+3xy-5y^2$
$-2A-2B=2x^2-14y^2$

$3B=6x^2+3xy+9y^2$

$B=2x^2+xy+3y^2$
$A+B=5x^2+2xy-y^2$

*예문에서 x^2,y^2의 계수가 모두 같고 xy계수만 다르므로 xy계수만 계산-> $xy=1$대입 후

$2A+B=3$ -①
$+A-B=0$ -②

$3A=3$->$A=1, B=1$
$A+B=2$

[정답] ④

12. 역함수와 합성 함수 성질을 확인하는 문제.

$f(x)=-x+2, y_{(x)}=2x+4$

$f\circ(g\circ f)^{-1}(10)=f\circ f^{-1}\circ g^{-1}(10)=g^{-1}(10)$

$g^{-1}(10)=a$ -> $g_{(a)}=10$ ->$a=3$
$g^{-1}(10)=3$

[정답] ③

13. 좌극한 우극한 성질.

우극한 $\lim_{x\to a}+f_{(x)}$ -> $x>a$

좌극한 $\lim_{x\to a}-f_{(x)}$ -> $x<a$

$$\left\langle \begin{matrix} \lim_{x\to -1}+f_{(x)}+\lim_{x\to 1}-f_{(x)} \\ x>1 \qquad \langle x<1\rangle \end{matrix}\right\rangle$$

$=1+1=2$

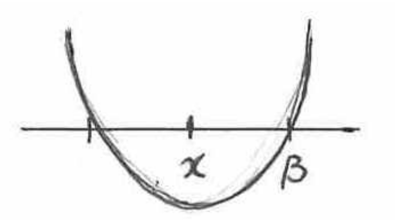
[정답] ②

14. 2차방정식이론.

$x^2=2kx+4=0$

$=f_{(x)}=x^2=2kx+4$

$=(x-k)^2-k^2+4$



$k^2+4\le 0, f_{(1)}=1-2k+4>0$

$k\ge 2\,\mathrm{or}\,k\le -2$

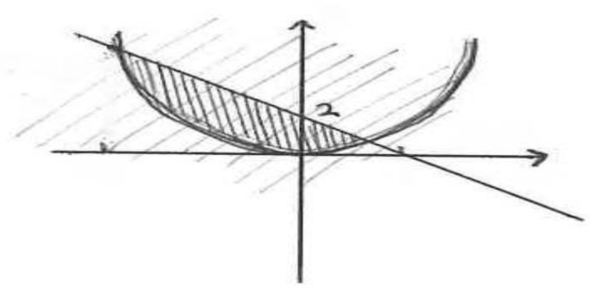
$$\left\langle \begin{matrix} k^2-4\ge 0 & k\ge 2\,\mathrm{or}\,k\le -2 \\ -2k+5>0 & k<\frac{5}{2} \end{matrix}\right\rangle$$

$\therefore 2\le k<\frac{5}{2}$

[정답] ④

15. $\begin{cases} y\ge x^2 \\ x+y-2\le 0 \end{cases} \rightarrow \begin{Bmatrix} x-y=k \\ \alpha-\beta(\text{최대}-\text{최소})= \end{Bmatrix}$

(1) 부등식 영역의 최대,최소 -> 그림 -> 영역내에서 평행이동 꼭지점, 접할 때 이루어진다.



(2) 교점 => $-x+2=x^2$

$$x^2+x-2=0$$

$(x+2)(x-1)=0$

$x=1 \text{ or } x=-2$

(1) $(-2,4) \to x-y=-6=k$ 최소값

(2) $\left\langle \begin{array}{l} y=x^2 \\ y=x-k \end{array} \right\rangle$

접할 때

$x^2-x+k=0$

$D=1-4k=0$

$$k=\frac{1}{4}$$

(3) (1.1) 지날 때 $x-y=0=k$

$\therefore -b \le x-y \le \dfrac{1}{4}$

$\alpha-\beta=\dfrac{1}{4}+b=\dfrac{25}{4}$

[정답] ③

부등식 역역=> 기울기 생각하면서 언제나 고정, 접할 때 **확인.**

16. 등차, 등비 중항 문제.

(1) $a,2,b \to$ 등차 $\dfrac{a+b}{2}=3$

$$a+b=6$$

(2) $\dfrac{\frac{1}{a}+\frac{1}{b}}{2}=\dfrac{3}{4}$ -> 등차

$\dfrac{1}{a}+\dfrac{1}{b}=\dfrac{3}{2}$

$\dfrac{a+b}{ab}=\dfrac{3}{2}$

$6\times 2=3ab \qquad ab=4$

$|a-b|^2=(a-b)^2=(a+b)^2=4ab$

$=36-16=20$

$|a-b|=2\sqrt{5}$

[정답] ①

<적용 공식>

등차 중항, $a\times b, x=\dfrac{a+b}{2}$

$|a-b|^2=(a-b)^2=(a+b)^2-4ab$

17. 우함수,기함수와 정적분 적용.

$\left\{\int_{-a}^{a} \text{우함수}\, dx = 2\left\{\int_0^a \text{우함수}\, dx\right\}\right\}$

$\left\{\int_{-a}^{a} \text{기함수}\, dx = 0 \text{ 활용}\right\}$

$\left\{\int_{-3}^{3} (x-1)f_{(x)dx}\right\}$

$=\left\{\int_{-3}^{3} xf_{(x)}dx - \left\{\int_{-3}^{3} f_{(x)}dx\right\}\right\}$

조건에 $f_{(x)}$ 우함수이므로 $xf_{(x)}$는 기함수

$\left\{ {}^{3}_{-3} xf_{(x)}dx = 0 \right\}$

$\therefore -2\left\{ {}^{3}_{0} f_{(x)}dx = -2\times(-2) \right\}$

=4

[정답] ②

18. 무리함수와 역함수 관계.

$y = 3\sqrt{x-2}+1$과 y^{-1}의 교정간 거리는

$y = 3\sqrt{x-2}+1$과 $y = x$의 교정거리와 같다.

$$\begin{cases} y = 3\sqrt{x-2}+1 \\ y = x \end{cases}$$

연립하면

$3\sqrt{x-2}+1 = x$

$3\sqrt{x-2} = x-1$

$9(x-2) = x^2 = 2x+1$

$x^2 - 11x + 19 = 0$

두근 α,β라면 $\alpha+\beta = 11 \quad \alpha\beta = 19$

두교점 (α,α) , (β,β) 이므로 두점간 거리

$|\alpha-\beta|^2 = (\alpha+\beta)^2 = 4\alpha\beta$

$= |2| - 76$

$= 45$

$|\alpha-\beta| = 3\sqrt{5}$

$\therefore$ 두점간거리 = $3\sqrt{10}$

[정답] ④

19. 확률.

A일 때 B일 확률은 $p = \dfrac{13}{A}$

$$\left\langle \begin{matrix} x+y+z = 12 \rightarrow \text{분모} \\ x = y = z \rightarrow \text{분자} \end{matrix} \right\rangle$$

$1 \le x \le 6, \quad 1 \le y \le 6, \quad 1 \le z \le 6$

$\therefore (1,5,6), (2,6,4), (2,5,5)$

$(3,4,5), (3,6,3), (4,4,4)$

$(1,5,6) \rightarrow 3 = 6$가지

$(2,6,4) \rightarrow 3 = 6$가지

$(3,4,5) \rightarrow 3 = 6$가지

$(2,5,5) \rightarrow \dfrac{3!}{2!} = 3$가지

$(3,6,3) \rightarrow \dfrac{3!}{2!} = 3$가지

$(4,4,4) \rightarrow 1$가지

$P = \dfrac{1}{25}$

[정답] ④

20. 근과 계수 + $\sum$계산.

$x^2 + 3nx + 2 = 0$ 두근 α_n,β_n

$\alpha_n + \beta_n = -3_n \qquad \alpha_n\beta_n = 2$

$\alpha_n^2 + \beta_n^2 = (\alpha_n+\beta_n)^2 - 2d_n\beta_n$

$= 9n^2 - 4$

$\sum_{n=1}^{5} \alpha_n^2 + \beta_n^2 \Rightarrow \sum_{n=1}^{5} (9n^2 - 4)$

$$=9\times\frac{5\times6\times11}{6}-4\times5$$

$$=45\times11-20$$

$$=475$$

[정답] ③

<적용 공식>

$$\left\langle \begin{array}{l} \sum_{k=1}^{n}k^2=\dfrac{n(n+1)(2n+1)}{6} \\ a^2+b^2=(a+b)^2-2ab \end{array} \right\rangle$$

$ax^2+bx+c=0$ 루근 α,β

$$\alpha+\beta=-\frac{b}{a},\quad \alpha\beta=\frac{c}{a}$$