

2014년 제2차 경찰공무원(순경) 수학 해설

01. ③ 02. ① 03. ② 04. ④ 05. ③ 06. ④ 07. ① 08. ③ 09. ① 10. ②
 11. ② 12. ④ 13. ② 14. ③ 15. ④ 16. ④ 17. ③ 18. ③ 19. ① 20. ②

1. 【정답】 ③

- ① $A \odot A = (A \cup A) \cap (A \cap A)^c = A \cap A^c = \phi$
 ② $B \odot \phi = (B \cup \phi) \cap (B \cap \phi)^c = B \cap U = B$
 ③ $B \odot U = (B \cup U) \cap (B \cap U)^c = U \cap B^c = B^c$
 ④ $A^c \odot B^c = (A^c \cup B^c) \cap (A^c \cap B^c)^c = (A \cap B)^c \cap (A \cup B) = A \odot B$

2. 【정답】 ①

$$a+5=5, a=0$$

$$A = \{-1, 2, 5\}, B = \{1, 3, 5\}$$

$$A \cup B = \{-1, 1, 2, 3, 5\}$$

$$-1+1+2+3+5=10$$

3. 【정답】 ②

$$a_n = n^2 + 3 - \{(n-1)^2 + 3\} = n^2 - (n-1)^2 = 2n-1 \quad (n \geq 2)$$

4. 【정답】 ④

$$\alpha \bar{\alpha} + \bar{\alpha} \beta + \alpha \bar{\beta} + \beta \bar{\beta} = \alpha(\bar{\alpha} + \bar{\beta}) + \beta(\bar{\alpha} + \bar{\beta}) = (\alpha + \beta)(\bar{\alpha} + \bar{\beta})$$

$$= (1-2i+1+3i)(1+2i+1-3i) = (2+i)(2-i) = 5$$

5. 【정답】 ③

$$-(x^2-1)(x^2+100) \geq 0$$

모든 실수 x 에 대하여 $x^2+100 > 0$ 이므로

$$-(x^2-1) \geq 0$$

$$x^2-1 \leq 0, (x-1)(x+1) \leq 0$$

$$-1 \leq x \leq 1$$

$x = -1, 0, 1$ 3개

6. 【정답】 ④

상수항을 비교하여 식을 다음과 같은 형태로 놓을 수 있다.

$$x^3 + ax + b = (x^2 - x + 1)(x + b + 2) + 3x - 2$$

$$(x^2 - x + 1)(x + b + 2) + 3x - 2 = x^2 + (b + 1)x^2 + (2 - b)x + b$$

$$b + 1 = 0, \quad b = -1$$

$$2 - b = 3 = a$$

$$a - b = 3 - (-1) = 4$$

7. 【정답】 ①

$$f(1)f(0) = f(1) + f(1)$$

$$-f(0) = -2, \quad f(0) = 2$$

$$f(1)f(1) = f(2) + f(0)$$

$$1 = f(2) + 2, \quad f(2) = -1$$

$$f(0) + f(2) = 2 - 1 = 1$$

8. 【정답】 ③

$$\text{두근의 곱 } \alpha\beta = \frac{1}{k-2} < 0, \quad k-2 < 0$$

$$k < 2$$

9. 【정답】 ①

$$f(x) = x^n - 10x^{n-1} + x^2 - x + 1 \text{이라 하면}$$

$$f(x) = (x - \alpha_1)(x - \alpha_2)(x - \alpha_3) \cdots (x - \alpha_n) \text{으로 놓을 수 있다.}$$

$$f(10) = (10 - \alpha_1)(10 - \alpha_2)(10 - \alpha_3) \cdots (10 - \alpha_n) \text{이므로}$$

$$f(10) = 10^n - 10 \cdot 10^{n-1} + 10^2 - 10 + 1 = 91$$

10. 【정답】 ②

5번째 경기에서 승리팀이 결정되려면 4번째 경기까지 A팀과 B팀이 2:2를 기록해야 한다.

따라서 확률은

$${}_4C_2 \left(\frac{2}{3}\right)^2 \left(\frac{1}{3}\right)^2 = \frac{6 \times 4}{3^4} = \frac{8}{27}$$

11. 【정답】 ②

$$x^3 - 1 = (x - 1)(x^2 + x + 1) = 0$$

$$\omega^3 = 1, \quad \omega^2 + \omega + 1 = 0$$

$$\omega^5 + \omega^4 + \omega^3 = \omega^3(\omega^2 + \omega + 1) = 0$$

12. 【정답】 ④

$$xy = 3$$

$$x + y \geq 2\sqrt{xy}$$

$$x + y \geq 2\sqrt{3}$$

13. 【정답】 ②

$$\frac{2+3+4+x+y}{5} = 4, \quad x+y = 11$$

$$\frac{(2-4)^2 + (3-4)^2 + (4-4)^2 + (x-4)^2 + (y-4)^2}{5} = 2$$

$$(x-4)^2 + (y-4)^2 = 5$$

$$x^2 - 8x + 16 + (7-x)^2 = x^2 - 8x + 16 + x^2 - 14x + 49 = 2x^2 - 22x + 65 = 5$$

$$2x^2 - 22x + 60 = 0, \quad x^2 - 11x + 30 = 0$$

$$(x-6)(x-5) = 0$$

$$\begin{cases} x=5 \\ y=6 \end{cases}, \quad \begin{cases} x=6 \\ y=5 \end{cases}$$

$$xy = 30$$

14. 【정답】 ③

$$2kx - x + ky + y + 2 - k = 0$$

$$(2x + y - 1)k + (-x + y + 2) = 0$$

k 에 대한 항등식이므로

$$2x + y - 1 = 0, \quad -x + y + 2 = 0$$

$$3x - 3 = 0, \quad x = 1, \quad y = -1$$

$$(1, -1)$$

15. 【정답】 ④

$$(\sin\theta + \cos\theta)^2 = 1 + 2\sin\theta\cos\theta = 2$$

$$\sin\theta\cos\theta = \frac{1}{2}$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1}{\sin\theta\cos\theta} = 2$$

16. 【정답】 ④

케일리-헤밀턴 정리에 의해

$$A^2 - (2+1)A + (2 \cdot 1 - 3 \cdot (-1))E = 0$$

$$A^2 - 3A + 5E = O$$

$$A^2 - 3A + 6E = E$$

$$A(A^2 - 3A + 6E) = A^3 - 3A^2 + 6A = A$$

$$A = \begin{pmatrix} 2 & 3 \\ -1 & 1 \end{pmatrix}$$

17. 【정답】 ③

$$(n+1)^3 + (n+2)^3 + (n+3)^3 + \dots + (2n)^3 = \sum_{k=1}^n (n+k)^3$$

$$(n+1)^4 + (n+2)^4 + (n+3)^4 + \dots + (2n)^4 = \sum_{k=1}^n (n+k)^4$$

따라서 준식은

$$\frac{4}{5} \lim_{n \rightarrow \infty} \frac{n \sum_{k=1}^n (n+k)^3}{\sum_{k=1}^n (n+k)^4} = \frac{4}{5} \lim_{n \rightarrow \infty} \frac{\sum_{k=1}^n \left(1 + \frac{k}{n}\right)^3 \frac{1}{n}}{\sum_{k=1}^n \left(1 + \frac{k}{n}\right)^4 \frac{1}{n}} = \frac{4}{5} \lim_{n \rightarrow \infty} \frac{\int_1^2 x^3 dx}{\int_1^2 x^4 dx} = \frac{4}{5} \times \frac{\frac{15}{4}}{\frac{31}{5}} = \frac{15}{31}$$

18. 【정답】 ③

$$\lim_{h \rightarrow 0} \frac{f(2+h) - f(2-h)}{3h} = \frac{1}{3} \lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} + \frac{f(2-h) - f(2)}{-h} = \frac{2}{3} f'(2)$$

$$\frac{2}{3} \times 9 = 6$$

19. 【정답】 ①

$$3x^2 = x^3 + 2x$$

$$x(x^2 - 3x + 2) = 0, \quad x = 0, 1, 2$$

$$0 < x < 1 \text{에서 } 3x^2 < x^3 + 2x \text{이고}$$

$$1 < x < 2 \text{에서 } 3x^2 > x^3 + 2x \text{이므로}$$

$$S = \int_0^1 (x^3 + 2x - 3x^2) dx + \int_1^2 (3x^2 - x^3 - 2x) dx$$

$$= \left[\frac{1}{4} x^4 + x^2 - x^3 \right]_0^1 + \left[x^3 - \frac{1}{4} x^4 - x^2 \right]_1^2 = \frac{1}{4} + 1 - 1 + 7 - \frac{15}{4} - 3 = \frac{1}{2}$$

$$2S = 1$$

20. 【정답】 ②

$$220 - 1.96 \frac{50}{\sqrt{100}} \leq m \leq 220 + 1.96 \frac{50}{\sqrt{100}}$$

$$220 - 1.96 \times 5 \leq m \leq 220 + 1.96 \times 5$$

$$210.2 \leq m \leq 229.8$$